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Determining consumer's carbon emission obligation through virtual emission tracing in power systems

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Abstract

Although a recent report by the U.S. Energy Information Administration indicates that CO_2 emissions from power generation have now fallen below emissions from the transportation sector, electricity use remains a significant part of human's environmental footprint since fossil fuels are the major energy resource for electricity generation worldwide. This explains the continual effort by policy makers to develop a remedy to mitigate carbon emissions. It is then critically important to properly account for the emissions associated with electricity generation, delivery, and consumption. In particular, it is critical to be able to allocate emissions to the relevant electricity consumers creating the demand that results in a polluting emission. Therefore, in this paper a *virtual carbon emission tracing method in power systems* is thoroughly presented to facilitate allotting carbon obligation. The method is developed based on the proportional sharing principle that has been used in power system operation. The proposed methodology is implemented on the IEEE 5-bus and 9-bus systems, two widely used and recognized power grid benchmarks.

KEYWORDS
carbon emissions, Power systems, proportional sharing principle, virtual emission tracing

1 | INTRODUCTION

The world-wide increase in anthropogenically sourced greenhouse gases (GHGs) over the past centuries has created severe negative impacts for humans and their environment, thereby promoting interest in carbon mitigation.[1-3] For example, the Kyoto Protocol, which is an international treaty to lower global CO_2 emissions is a critical effort to address this crucial issue.[4] A considerable number of research studies have been conducted in recent years on low-carbon technologies. However, aside from the technical aspects of carbon mitigation, the methodology for evaluating the responsible source and amount of emissions is also of vital importance in low-carbon development and for setting up appropriate emission-related policies.[3,5-7] Recent literature suggests basically two approaches for addressing responsibility for CO_2 emissions. One says that the emission producer (generator) is responsible and the second suggests that it is the responsibility of the consumer of the generated power that creates the emission.[4] The principle presumed in the Kyoto Protocol is the former one, that is, the production accounting principle. The absence of liability on the consumer side is an important disadvantage of such a principle.[8] This absence provides an important impetus for international CO_2 accounting standards.[4,7] That is why an increasing number of studies have lately asked for attention to the fact that “consumers” should also be responsible when it comes to CO_2, rather than “producers” alone.[4,9-11] Reference 12 claims, as well, that carbon emission embodied in electricity trade is essential for responsibility allocation between producers and consumers. This is in line with the consumption accounting principle, which is also the approach adopted in this paper.

The large-scale connectivity between electricity producers and consumers has required the development of vast energy transmission networks. Although there can be significant differences (e.g., in fuel type, efficiency, generation technology, emission factor, etc.) between these production/generation units, the delivered product, that is, the electricity, is identical throughout the power system.[12] Thus, it is

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challenging to attribute the source of electricity consumed at any point and time.[13] A useful concept to assist in this effort is "virtual carbon flow" which accompanies the energy flow in a power system. This paper describes and develops "virtual carbon tracing in power systems" as a method to determine carbon emission flow in power networks.[4] The way carbon emissions are dealt with in the rest of this paper differs from the way in which they are usually treated. We model carbon *flows* in power systems as "virtual carbon flow" rather than representing carbon emissions at static points in the system. By considering the virtual carbon flows, carbon emission obligation is more tangibly represented.

At present, general research on carbon flow in power systems is still in its early stages. This paper provides new insights into the carbon tracing concept and is based on linear algebra analysis. Most importantly, the proposed carbon tracing method depends on the use of distribution matrices whose mathematical proof and invertibilities are already demonstrated in the literature of the power tracing. Furthermore, the aforesaid matrices are based on a proportional sharing principle which is the most widely used method for power tracing in power system operation. The proposed carbon tracing method is quite transparent and practical, and can be readily implemented on any system regardless of its size due to the efficiencies in the computational method. Additionally, the utilized power tracing methodology is capable of handling circulated power flows and also the power losses in transmission lines. Last but not least, we introduce readers with an interesting way to gather the data needed for the purpose of performing carbon tracing in case the required information is not publicly available.

The authors believe this paper may provide policy makers with a more tangible understanding of carbon emission in power systems. Additionally, the methodology presented in this paper can help power utilities improve system design and operation to minimize carbon emissions.

The remainder of the paper is organized as follows: In the next section, a power flow tracing method widely used in power system operation is reviewed and the proposed virtual carbon emission tracing technique is then presented followed by two case studies. A novel approach to collect data necessary for carbon tracing is then introduced. Finally, closing remarks are provided.

2 | VIRTUAL EMISSION TRACING

In power systems, a virtual carbon emission flow can be assumed tightly integrated with the power flow. The emission flows along a path identical to the power flow but in an opposite direction. The concept is initially developed through the simplistic hypothetical case shown in Figure 1 consisting of a single customer–single producer. The power producer is a thermal generating unit. The generator emits a certain amount of carbon pollution (emissions). We may consider a hypothetical flow of carbon leaving the consumer and moving toward the generator along the same path as the power flow (Figure 1). The carbon pollution is physically sourced (emitted) at the generation site. This means that the consumer receives some service or product in the form of electricity whose waste, in the form of carbon emission, is then sent back upstream to the generation site. This virtual carbon emission travels at the same speed as the electrical current.

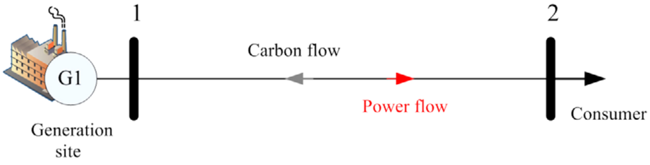

FIGURE 1 A single customer–single producer system [Colour figure can be viewed at wileyonlinelibrary.com]

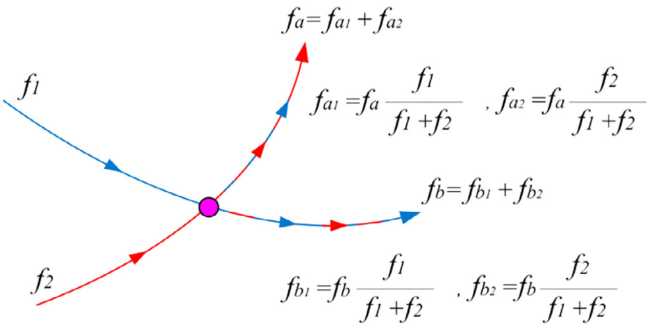
$f_a = f_{a1} + f_{a2}$
f_1
$f_{a1} = f_a \frac{f_1}{f_1+f_2}$
$, f_{a2} = f_a \frac{f_2}{f_1+f_2}$
$f_b = f_{b1} + f_{b2}$
f_2
$f_{b1} = f_b \frac{f_1}{f_1+f_2}$
$, f_{b2} = f_b \frac{f_2}{f_1+f_2}$

FIGURE 2 Proportional sharing principle [Colour figure can be viewed at wileyonlinelibrary.com]

As demonstrated in Figure 1, the basic application of this proposed technique is very simplistic. However, real applications are not as simple as this, since real-life power networks are much more sophisticated and the determination of the distribution of the virtual carbon emission flows requires parametrical power tracing. Then, by tracing back loads to generation and applying the relationship between emission and energy production for different generators, virtual carbon emission flows will be determined. This allows assigning the carbon mitigation obligations and enables the system operators to identify the efficiency of carbon utilization for the whole network.[3]

Through power tracing, one can compute how much power travels from a specific generator to a specific load point as well as determine to what degree a load or generator contributes to the active power flow in a branch.[14,15] In the traditional power system structure (vertically integrated), these computations were of little importance. However, in a restructured environment, calculating the aforesaid contributions are imperative for transmission cost allocation.[16]

Power flow tracing was first introduced in 1990.[17] Methods to implement power tracing can be categorized as numerical and graphical. Numerical methods benefit from matrix computation and are of simple algorithms. There are three key components necessary to use numerical methods[18]: (a) a snapshot of a steady-state solution of the system must be available; (b) a generator connected at bus *i* is prior to any other generator to supply the demand on bus *i*; and (c) the flows of electricity obey the proportional-sharing rule (as shown in Figure 2).[19]

There are different methods given in the literature to perform power tracing.[14-16,18-22] However, only a few of them are capable of dealing with circulating flows and power losses in lines. One main reason for power cycle is the use of phase angle shifting devices. In this paper, we adopt the power tracing methodology given in Reference 21 as it not only accounts for the influence of transmission losses but also can be used for systems with and without circulating flows; and more importantly, it is quite transparent. A brief description of this method is given in the following. It should be noted that in this paper we only deal with the active term of power flow—for two reasons. First, for a thermal generator, fuel consumption rate remains reluctant against change in reactive power output. Second, reactive power can be locally generated using passive devices.

For the power tracing method given in Reference 21, there are two important matrices referred to as A_u and A_d, downstream and upstream distribution matrices, respectively. They are calculated as defined by Equations (1) and (2):

$$A_d = I + B_d^T.\text{diag}(F).B_u.[\text{diag}(P)]^{-1}, \tag{1}$$

$$A_u = I + B_u^T.\text{diag}(F).B_d.[\text{diag}(P)]^{-1}. \tag{2}$$

Where B ($B = B_u + B_d$) is the branch-node incidence matrix of the power network under study. $B(b, n)$ is 1 (−1) if the bth branch is a line leaving (entering) nth node, respectively (as defined by the direction of the flow). Otherwise, it is zero. B can be split into matrix B_u consisting of entries of (−1) and B_d consisting entries of (1).

Although the topic is very important, a discussion of steady-state analysis of power systems including economic dispatch and power flow equations (optimal power flow) is not included in this paper for the sake of brevity.

After performing the power flow calculation, F, B_u, and B_d will be known, and hence, A_u and A_d can be calculated. Once A_u and A_d are known, the contribution matrices can be extracted as well. Equations (3) and (4) have been introduced in[21] to derive contributions of loads to generators and line flows, respectively. However, these two equations do not provide the matrices needed to carry out a “virtual emission tracing”. Instead, the emission tracing requires contribution matrices of generators to loads and generators to line flows.

$$\boldsymbol{P_{Gi}} = \frac{P_{Gi}}{P_i}\sum_{k=1}^{nbus}\left[A_d{}^{-1}\right]_{i,k}.\boldsymbol{P_{Dk}}, \tag{3}$$

$$\boldsymbol{P_{ij}} = \frac{P_{ij}}{P_i}\sum_{k=1}^{nbus}\left[A_u{}^{-1}\right]_{i,k}.\boldsymbol{P_{Dk}}. \tag{4}$$

Reference 22 states that for a given node, if the proportion of the inflow, which can be traced to generator i, is α_i, then the contribution of the load, which is connected to the same node, is also α_i. Regarding this fact and Reference 21's method, the following two equations are introduced by the authors to compute the generators' contributions to loads and line flows

$$\boldsymbol{P_{Di}} = \frac{P_{Di}}{P_i}\sum_{k=1}^{nbus}\left[A_u{}^{-1}\right]_{i,k}.\boldsymbol{P_{Gk}}, \tag{5}$$

$$\boldsymbol{P_{ij}} = \frac{P_{ij}}{P_i}\sum_{k=1}^{nbus}\left[A_u{}^{-1}\right]_{i,k}.\boldsymbol{P_{Gk}}. \tag{6}$$

Where P_i is the nodal inflow power at the source end of P_{ij} (assuming that P_{ij} represents power flowing from node i to j). The proofs of Equations (5) and (6) are very simple. Equation (7) is given in Reference 21 for determining generators' contributions to the nodal power vector (P).

$$P = A_u^{-1}P_G. \tag{7}$$

Equation (7) can be rewritten as below for an arbitrary row of matrix P. Therefore,

$$\boldsymbol{P_i} = \sum_{k=1}^{nbus}\left[A_u{}^{-1}\right]_{i,k}P_{Gk}. \tag{8}$$

Multiplying both sides of Equation (8) by P_{Di} and then manipulating it, results in Equation (5). Similarly, Equation (6) is obtained if Equation (8) is multiplied by P_{ij}.

As stated in Reference 21, transmission losses can be accounted for by considering gross and net network flows. To obtain the net flows, the vector F used to calculate A_d in Equation (1) must be replaced with the vector of flows at the receiving end of each line. To obtain the gross flows, the flow vector F used to calculate A_u in Equation (2) must be replaced with the vector of flows at the source end of each line. This will be further discussed in the following section.

3 | SIMULATION RESULTS AND DISCUSSIONS

In this section, the proposed virtual emission tracing is implemented on two well-known standard power grids. The technical information of these two case studies (i. e., the topology and the system state of the power network) is readily available and given in the literature. However, these data are considered confidential, at least in part, and not publicly available when it comes to real-life networks. Therefore, one might ask how the proposed carbon tracing model can be applied to real-world networks as decision makers may not have access to such data. This issue will not be a problem if the proposed analysis is done by independent system operators (ISOs) or regional transmission organizations (RTOs), as they are privy to all the necessary data. Even if ISOs/RTOs are neither willing to carry out the analysis nor allowed to provide the necessary data to perform the analysis, there still may be some viable approaches for performing the analysis in practice. For example, Genscape[23] is a company providing real-time data of power grids, including power flows, which can be used for carbon tracing. Genscape has established a platform (PowerRT) to provide clients with the real-time status of power grids and help them optimize revenues, minimize risks and even forecast market trends. By the use of this platform, any issues associated with access to high-resolution power plant output and transmission flow data will be alleviated. The platform is also capable of calculating in real time how much fuel is being consumed. Nevertheless, in this paper we are applying the proposed method to standard test systems, meaning that all the required data are publicly available. The first case study we consider in this paper is the IEEE 5-bus test system. A given optimal power flow snapshot of this network is shown in Figure 3. This snapshot is obtained from MATPOWER.[24]

The two previously mentioned binary matrices, B_u, B_d, for this network will be as follows:

$$(-1)\times B_u=\begin{pmatrix}0&1&0&0&0\\0&0&0&1&0\\1&0&0&0&1\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\end{pmatrix}\quad B_d=\begin{pmatrix}1&0&0&0&0\\1&0&0&0&0\\0&0&0&0&1\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}.$$

As mentioned earlier, power flow calculation is out of the scope of this paper. Therefore, Tables 1 and 2 briefly present the results of the steady-state analysis (power flow) of the 5-bus network.

The two types of line flows, gross and net, which were already explained in the preceding section, are given in Table 1.

Like the assumption of Reference 11 each of the generators in the system shown in Figure 3 is assumed to be one of the three types given in Table 3. Those with capacities greater than 330 MW are considered coal-fired and those with capacities less than 100 MW are defined as zero-carbon emission units (i.e., hydro, wind, and solar power plants). The remaining generators are considered gas-turbine generators. There are two hypotheses in regard to Table 3 that should be mentioned. First, although thermal units emit different kinds of emission, only CO_2 has been counted in Table 3, as it is the foremost GHG pollutant. Second, for the sake of simplicity, the emission rates given in Table 3 are assumed constant. In practice, they may depend on the power output of generators.[25] Nevertheless, this does not affect the methodology presented in this paper.

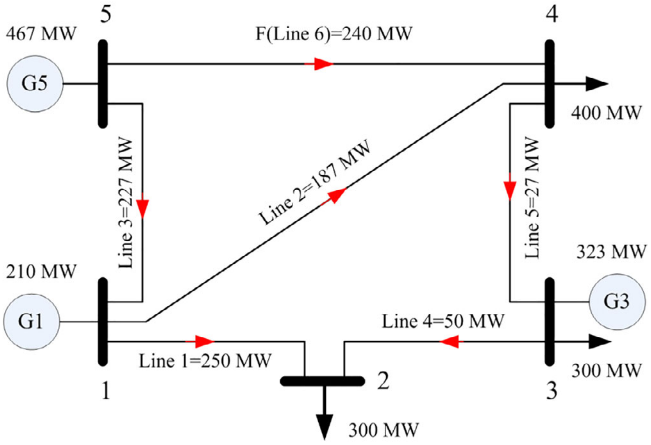

FIGURE 3 The IEEE 5-bus test system with DC optimal power flow results [Colour figure can be viewed at wileyonlinelibrary.com]

Equations (9)–(11) give the contribution factors of the generators to the loads. Equations (9)–(11) are obtained by applying Equation (5) to a given state (Tables 1 and 2) of the system which was shown in Figure 3. Likewise, Equations (12–17), which give the contributions of the generators to the line flows, are obtained if Equation (6) is applied to the system state.

$$P_{D2}=0.57P_{G1}+0.14P_{G3}+0.29P_{G5}, \tag{9}$$

$$P_{D3}=0.03P_{G1}+0.86P_{G3}+0.04P_{G5}, \tag{10}$$

$$P_{D4}=0.4P_{G1}+0.0P_{G3}+0.67P_{G5}, \tag{11}$$

$$F_1=P_{12}=0.57P_{G1}+0.0P_{G3}+0.28P_{G5}, \tag{12}$$

$$F_2=P_{14}=0.43P_{G1}+0.0P_{G4}+0.21P_{G5}, \tag{13}$$

$$F_3=P_{51}=0.0P_{G1}+0.0P_{G3}+0.49P_{G5}, \tag{14}$$

$$F_4=P_{32}=0.0P_{G1}+0.14P_{G3}+0.01P_{G5}, \tag{15}$$

$$F_5=P_{43}=0.03P_{G1}+0.0P_{G3}+0.05P_{G5}, \tag{16}$$

$$F_6=P_{54}=0.0P_{G1}+0.0P_{G3}+0.51P_{G5}. \tag{17}$$

Since the contribution factors of the generators to the demand and line flows are now available, we use the CO_2/kWh rates given in Table 3 to calculate the actual values of the carbon obligations of load points and lines. The results are visually displayed in Figures 4 and 5.

TABLE 1 Line flow data determined from power flow analysis on the IEEE 5-bus network

From node	To node	*X* (p.u.)	*R* (p.u.)	Gross power flow (MW)	Net power flow (MW)	DC power flow (MW)
1	2	0.00281	0.0281	249.8	248	250
1	4	0.00304	0.0304	186.5	185.4	187
5	1	0.00064	0.0064	226.6	226.3	227
3	2	0.00108	0.0108	52.1	52	50
4	3	0.00297	0.0297	28.7	28.6	27
5	4	0.00297	0.0297	239.9	238.2	240

As the carbon emission flowing across the network is now known, we may calculate the carbon intensity of branches and nodes. The branch carbon intensity can be defined as the ratio of the amount of carbon emission flow to the corresponding power flow. Similarly, the nodal carbon intensity is defined as the sum of all the branch carbon emissions entering a given node divided by the nodal power of the same node. Therefore,

$$CIN_k = \left(\sum_{\substack{i=1 \\ i \neq k}}^{nbus} E_{ik} \right) / P_k, \tag{18}$$

$$CIB_{ij} = \frac{E_{ij}}{P_{ij}}. \tag{19}$$

These two criteria (CIN_k and CIB_{ij}) facilitate sorting the branches and nodes of the network according to the level of emissions, thereby indicating to what degree a line or bus is contributing to pollution. This is shown in Figure 6 for the 5-bus system.

TABLE 2 Load/generation data

Node	Load [MW]	DC generation [MW]	Maximum capacity of G_i (MW)
1	–	210	250
2	300	–	–
3	300	323	330
4	400	–	–
5	–	467	600

TABLE 3 Carbon emission rates of the thermal generators in the system under study[11]

Unit type	Capacity (MW)	Emission rate (kgCO_2/kWh)
Coal-fired	C > 330	0.875
Gas-turbine	100 < C < 330	0.500
Zero-emission	C < 100	0.000

As expected, it is clearly observed in Figure 6 that the carbon emission intensity of a given line (or bus) is close to the emission rate of the generator whose contribution to the same line (or bus) is dominant. For example, as seen in Figure 6, the *CIN* at node 3 is 0.52; very close to 0.50, the emission rate of the generator connected to it (Table 3).

It is interesting and illuminating to note that Equation (19) can be substituted into Equation (18) to get Equation (20). Equation (20) yields the fact that nodal carbon emission intensity (CIN_k) is a weighted average of the carbon intensities of the lines involved.

$$CIN_j = \left(\sum_{\substack{i=1 \\ i \neq j}}^{nbus} CIB_{ij}.P_{ij} \right) / P_j. \tag{20}$$

In a manner similar to Figure 4 for the 5-bus system, Figure 7 displays the results of applying the proposed carbon tracing method to

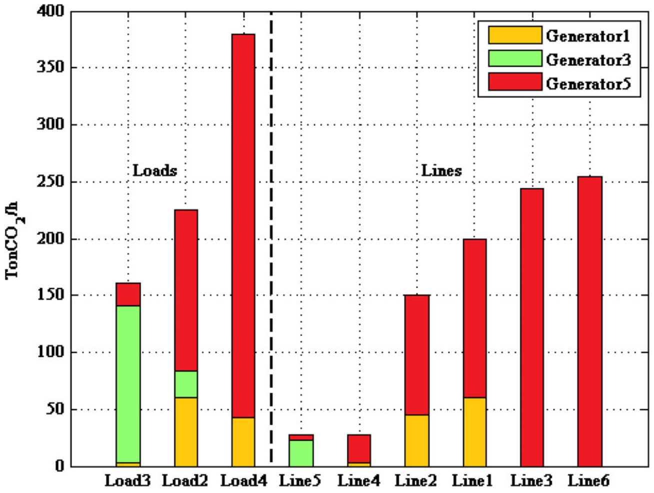

FIGURE 5 Carbon obligation of loads along with the virtual CO_2 passing through lines [Colour figure can be viewed at wileyonlinelibrary.com]

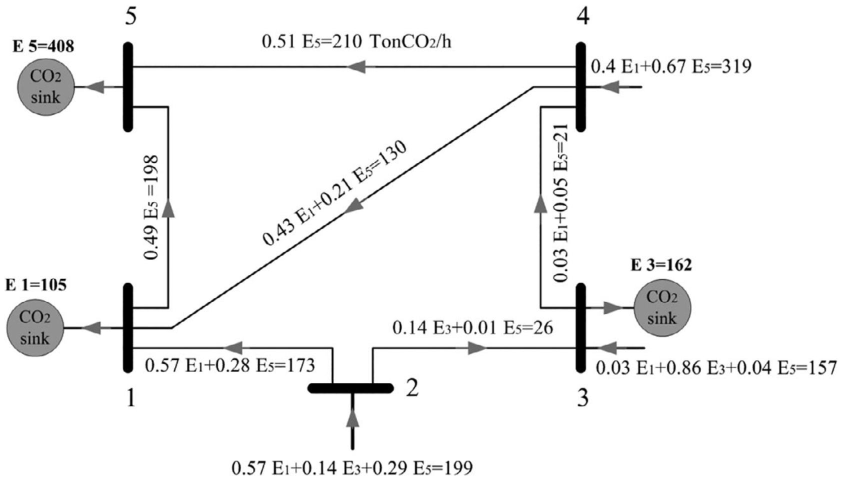

FIGURE 4 The results of virtual emission tracing for the 5-bus case study

the IEEE 9-bus power system. In this application, it was assumed that the emission rate of all the generators is the same and equal to 0.50 kgCO$_2$/kWh. The basic information of this power network is given in Reference 24 and therefore, not provided here for the sake of brevity.

Figure 8 briefly displays the procedure to implement the proposed methodology.

4 | CLOSING REMARKS

A comprehensive virtual carbon tracing model was introduced in this paper. The method started with the determination of the contribution factors of generators to loads and power passing through transmission lines of a power system. These factors were then used to define a virtual carbon flow in power networks. This virtual carbon emission flow is hypothetical, meaning that it does not physically travel between the loads and generators. However, it is capable of quantifying as well as visually showing the carbon obligations of loads and transmission lines. In order to exemplify and showcase how the proposed method can be employed, it was implemented on two well-known benchmark power networks (i. e., IEEE 5-bus and IEEE 9-bus test systems) followed by the analysis of the numeric results. More importantly, two indices were introduced to calculate the nodal and line carbon intensities.

The proposed method involves small computational effort and can facilitate real-time processing thanks to its lack of need for searching or adding dummy nodes to account for line losses. Additionally, the method effectively helps us improve our understanding of both the emission levels of a region and how to encourage demand-side measures to prompt emission reduction as the carbon obligations of loads (consumers) are now quantified.

The proposed method also clearly quantifies the embodied carbon associated with electricity consumed at different locations and times. With this knowledge, policy makers can set up the appropriate strategies to ensure that large-scale consumers (or industries such as gas and water utilities capable of doing spatial load management[26,27]) move demands in space and time to responsibly manage carbon emissions. This method will also prove beneficial for the development of appropriately sized and located renewable energy units. The method can also be promising for environmentally friendly, and therefore, sustainable network expansion planning and asset management. For example, a transmission line that generally carries electricity of low carbon intensity should be prioritized when power network upgrades occur. Moreover, by relying on the proposed method, price signals of

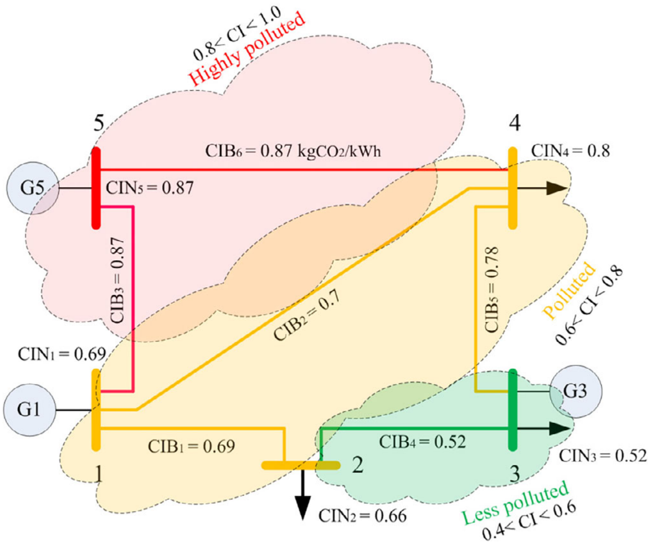

FIGURE 6 The IEEE test system labeled with carbon tracing results [Colour figure can be viewed at wileyonlinelibrary.com]

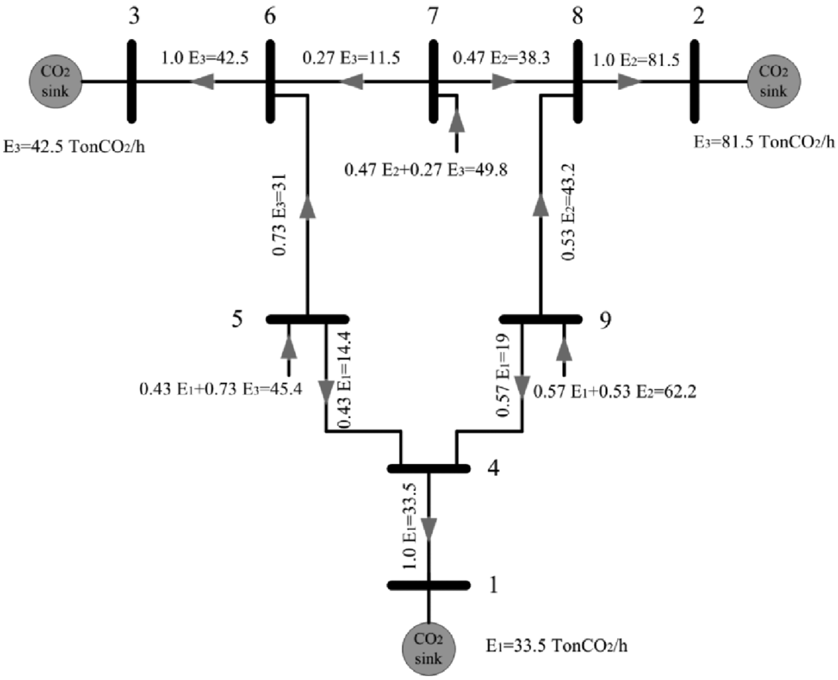

FIGURE 7 The results of virtual emission tracing for the 9-bus case study

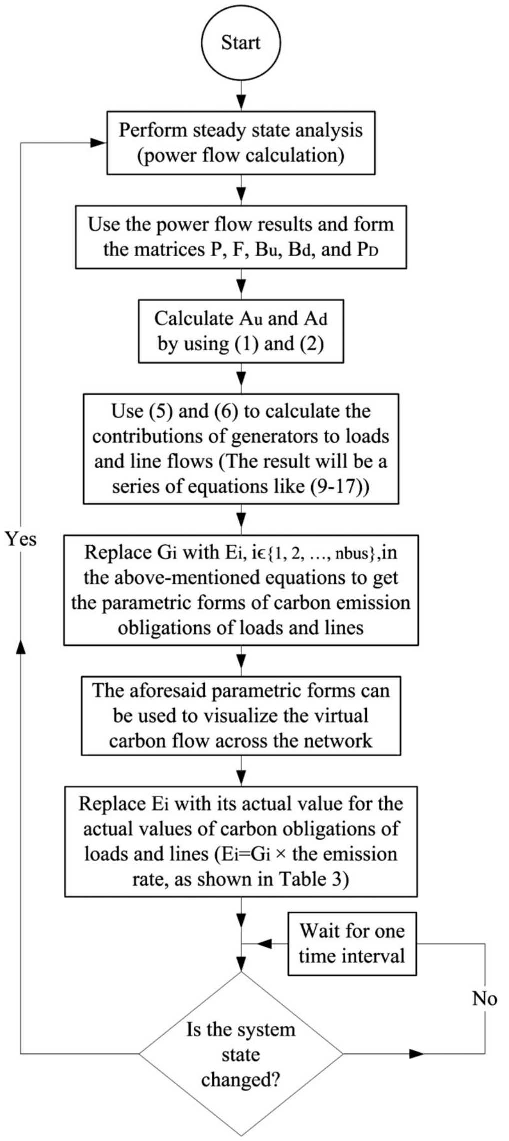
Start
Perform steady state analysis (power flow calculation)
Use the power flow results and form the matrices P, F, Bu, Bd, and PD
Calculate Au and Ad by using (1) and (2)
Use (5) and (6) to calculate the contributions of generators to loads and line flows (The result will be a series of equations like (9-17))
Replace Gi with Ei, i∊{1, 2, …, nbus},in the above-mentioned equations to get the parametric forms of carbon emission obligations of loads and lines
The aforesaid parametric forms can be used to visualize the virtual carbon flow across the network
Replace Ei with its actual value for the actual values of carbon obligations of loads and lines (Ei=Gi × the emission rate, as shown in Table 3)
Wait for one time interval
Is the system state changed?
Yes
No

FIGURE 8 The flowchart of the proposed carbon tracing method

carbon emission can be easily integrated with nodal electricity prices thereby allowing consumers to mutually optimize their consumption for both cost savings and carbon mitigation. An additional, and significant, benefit of the proposed method is the capability to quantify the carbon embodied in goods and services. This is traditionally neglected (or only approximately considered) in life-cycle analysis and carbon footprint accounting. This is critical to the development of a future carbon-trading framework and an understanding of embodied carbon, which is an emerging research topic.[28-30] The authors offer the term “Energy-Emission Transparency” to describe the determination of spatially and temporally variable carbon associated with 1 watt-hour of energy at different buses of a power grid. This aspect of exchange between producer and consumer demonstrates how electricity-importing regions (municipalities, states, countries) may covertly impose their carbon reduction obligation to the exporting region without being charged. One more promising advantage of the proposed framework is to give assistance to environmental advocates when it comes to right of way. A right-of-way corridor for a power transmission line is the land that a power utility occupies to construct or maintain overhead power lines. Sadly, the corridor allows the utility to disturb the natural environment and provide clearance from trees that could interfere with the line installation procedure, and future maintenance and operation. The proposed methodology grabs the policy makers and official environmentalists the freedom to ask the power utility (Transco) for the expected carbon intensity (ECI) of the new power transmission line. ECI can play the role of a formal and tangible criterion of how the new power line is going to be clean or polluted. In this way, they can easily determine if installing the new power line is worth doing the mechanical line clearance activities, and therefore, have the higher hand when negotiating over the corridor as the environment matters.

ACKNOWLEDGMENT

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NOMENCLATURE

Indices and Abbreviations

b	Shortened form of branch
d	Index for downstream
CI	Carbon intensity ($kgCO_2/kWh$)
CIB	Carbon intensity of branch ($kgCO_2/kWh$)
CIN	Carbon intensity of node ($kgCO_2/kWh$)
D	Index for demand
diag	Diagonal matrix (The entries outside the main diagonal are all zero)
E	Emission rate ($TonCO_2/hour$)
f_1, f_2(f_a, f_b)	The input (output) power flows
G	Index for generation
h	Shortened form of hour
i, *j*, *k*	Indices for the nodes of the water distribution network
n	Shortened form of node
nbus	Number of buses
R	The real term of the complex impedance of a transmission line (p.u.)
u	Index for upstream
X	The imaginary term of the complex impedance of a transmission line (p.u.)

Matrices and Vectors

A	Power network distribution matrix
B	The branch-node incidence matrix (elements are 1, -1, and zero)
F	The vector of branch flows
I	The identity (or unit) matrix (of size *nbus* ×*nbus*, ones on the main diagonal and zeros elsewhere)

P	Vector of nodal flows (the sum of nodal inflows/outflows including local generation/demand of nodal inflows or outflows including local generation and demand, respectively),
P_D	Vector of nodal demands
P_G	Vector of nodal generations

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